SAFE HANDS & IIT-ian's PACE

LEAP TEST# 01 (JEE) ANS KEY Dt. 29-11-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	D	31	C	61	A
2	C	32	C	62	B
3	AC	33	A	63	D
4	D	34	A	64	B
5	A	35	C	65	A
6	D	36	A	66	D
7	D	37	C	67	D
8	D	38	A	68	D
9	B	39	A	69	A
10	B	40	C	70	A
11	A	41	D	71	D
12	B	42	D	72	A
13	B	43	B	73	D
14	D	44	D	74	B
15	D	45	A	75	B
16	A	46	A	76	A
17	BONUS	47	D	77	D
18	D	48	C	78	C
19	D	49	C	79	B
20	D	50	B	80	A
21	3	51	2	81	1
22	3	52	4.5	82	45
23	3	53	158	83	5
24	40.8	54	0	84	4
25	0	55	2	85	1

SAFE HANDS & PACE

LT#01 (JEE) MATHS SOLUTIONS

: ANSWER KEY :

61)	a	62)	b	63)	d	64)	b
65)	a	66)	d	67)	d	68)	d
69)	a	70)	a	71)	d	72)	a
73)	c	74)	b	75)	b	76)	a
77)	d	78)	c	79)	b	80)	a

81)	1	82)	45	83)	5	84)	4
85)	1						

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(a)**

We have,

$\sum a^3 \cos(B-C)$

$= \sum k^3 \sin^3 A \cos(B-C)$

$= k^3 \sum \sin^2 A \sin(B+C) \cos(B-C)$

$= \frac{k^3}{2} \sum \sin^2 A(\sin 2B + \sin 2C)$

$= \frac{k^3}{2} \sum [\sin^2 A(\sin 2B + \sin 2C)$

$+ \sin^2 B(\sin 2C + \sin 2A) + \sin^2 C(\sin 2A + \sin 2B)$

$= k^3 \sum [\sin^2 A \sin B \cos B + \sin^2 B \sin A \cos A]$

$= k^3 \sum \sin A \sin B \sin(A+B)$

$= k^3[\sin A \sin B \sin C + \sin B \sin C \sin A$

$+ \sin C \sin A \sin B]$

$= 3(k \sin A)(k \sin B)(k \sin C) = 3\,abc$

62 **(b)**

We have,

$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

$\Rightarrow \frac{\sin(x+y)+\sin(x-y)}{\sin(x-y)-\sin(x-y)} = \frac{(a+b)+(a-b)}{(a+b)-(a-b)}$

$\Rightarrow \frac{2\sin x \cos y}{2\cos x \sin y} = \frac{2a}{2b}$

$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$

64 **(b)**

Given equations can be rewritten as

$\cos\theta = \frac{a}{x-h}, \qquad \sin\theta = \frac{b}{y-k}$

Now, $\frac{a^2}{(x-h)^2} + \frac{b^2}{(y-k)^2} = 1 \quad [\because \cos^2\theta + \sin^2\theta = 1]$

65 **(a)**

$p = \sin^2 x + \cos^2 x\,(1-\sin^2 x)$

$\Rightarrow p = (\sin^2 x + \cos^2 x) - \sin^2 x \cos^2 x$

$= 1 - \sin^2 x \cos^2 x \quad$ ...(i)

Which shows $p \le 1$

Again, $p = 1 - \cos^2 x + \cos^4 x$

$p = \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$

Which shows

$p \ge \frac{3}{4} \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{3}{4} \le p \le 1$

6 **(d)**

Given, $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2}-1\right) + \log(\cos x)$

Here, 4^{-x^2} is defined for $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$, $\cos^{-1}\left(\frac{x}{2}-1\right)$ is defined,

If $-1 \le \frac{x}{2} - 1 \le 1 \Rightarrow 0 \le x \le 4$

And $\log(\cos x)$ is defined, if $\cos x > 0$

$\Rightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$

Hence, $f(x)$ is defined for $x \in \left[0, \frac{\pi}{2}\right]$

67 **(d)**

Given, $f: C \to R$ such that $f(z) = |z|$

We know modulus of z and $\bar{z}$ have same values, so $f(z)$ has many one.

Also, $|z|$ is always non-negative real numbers, so it is not onto function.

68 **(d)**

$f(x) = \text{cosec}^2 3x + \cot 4x$

Period of $\text{cosec}^2 3x$ is $\frac{\pi}{3}$ and $\cot 4x$ is $\frac{\pi}{4}$.

$\therefore$ Period of $f(x) = \text{LCM of} \left\{\frac{\pi}{3} \text{ and } \frac{\pi}{4}\right\}$

$= \frac{\text{LCM of } (\pi, \pi)}{\text{HCF of } (3,4)} = \frac{\pi}{1} = \pi$

69 **(a)**

Since $\sqrt{\cos(\sin x)}$ exists for all $x \in R$ and $\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ exists for $x = \pm 1$. Therefore,

$f(x) = \sqrt{\cos(\sin x)} + \sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined for $x \in [-1, 1]$

70 **(a)**

Let x and y be two arbitary elements in A.

Then, $f(x) = f(y)$

$\Rightarrow \quad \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$\Rightarrow x = y, \forall\, x, y \in A$

So, f is an injective mapping.

Again, let y be an orbitary element in B, then

$f(x) = y$

$\Rightarrow \quad \frac{x-2}{x-3} = y$

$\Rightarrow \quad x = \frac{3y-2}{y-1}$

Clearly, $\forall\, y \in B, x = \frac{3y-2}{y-1} \in A$, thus for all $y \in B$ there exists $x \in A$ such that

$$f(x) = f\left(\frac{3y-1}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = y$$

Thus, every element in the codomain B has its preimage in A, so f is a surjection. Hence, $f: A \to B$ is bijective.

71 **(d)**

The given function is

$f(x) = |x| = \begin{cases} x, x \geq 0 \\ x, x < 0 \end{cases}$

And $f: R \to R$, then it is clear that function is neither one-one nor onto.

72 **(a)**

Let $y = f(x) = x^3$

$\therefore \quad x = y^{1/3}$

$\Rightarrow \quad f^{-1}(x) = x^{1/3}$

$\therefore \quad f^{-1}(8) = (8)^{1/3} = 2$

73 **(c)**

We have,

$f(x) = 6^x + 6^{|x|} > 0$ for all $x \in R$

$\therefore$ Range $(f) \neq$ (Co $-$ domain (f)

So, $f: R \to R$ is an into function

For any $x, y \in R$, we find that

$x \neq y \Rightarrow 2^x \neq 2^y \Rightarrow 2^{x+|x|} \neq 2^{y+|y|} \Rightarrow f(x) \neq f(y)$

So, f is one-one

Hence, f is a one-one into function

75 **(b)**

Let $y = f(x) = 2^{x(x-1)}$

$\Rightarrow \quad \log_2 y = x^2 - x \Rightarrow x^2 - x - \log_2 y = 0$

$\Rightarrow \quad x = \frac{1 \pm \sqrt{1 + 4\log_2 y}}{2} = \frac{1 + \sqrt{1 + 4\log_2 y}}{2}$

$\left[\because x = \frac{1 - \sqrt{1 + 4(x^2 - x)}}{2} = \frac{1 - (2x - 1)}{2} < 0 \text{ domain is not defined}\right]$

76 **(a)**

We have,

$A \, \Delta \, B = (A \cup B) - (A \cup B)$

$\Rightarrow n(A \, \Delta \, B) = n(A) + n(B) - 2\, n(A \cap B)$

So, $n(A \, \Delta \, B)$ is greatest when $n(A \cap B)$ is least

It is given that $A \cap B \neq \phi$. So, least number of elements in $A \cap B$ can be one

$\therefore$ Greatest possible value of $n(A \, \Delta \, B)$ is $7 + 6 - 2 \times 1 = 11$

77 **(d)**

We have,

$\frac{n^3 + 5n^2 + 2}{n} = n^2 + 5n + \frac{2}{n}$

$\therefore \frac{n^3+5n^2+2}{n}$ is an integer, if $\frac{2}{n}$ is an integer

$\Rightarrow n = \pm 1, \pm 2$

$\Rightarrow$ A consists of four elements viz. $-1, 1, -2, 2$

78 **(c)**

Since R is a reflexive relation on A.

$\therefore (a, a) \in R$ for all $a \in A$

$\Rightarrow n(A) \leq n(R) \leq n(A \times A) \Rightarrow 13 \leq n(R) \leq 169$

79 **(b)**

Let $(a, b) \in R$. Then,

a and b are born in different months $\Rightarrow (b, a) \in R$

So, R is symmetric

Clearly, R is neither reflexive nor transitive

80 **(a)**

Let $\mathcal{U}$ be the set of all students in the school. Let C, H and B denote the sets of students who played cricket, hockey and basketball respectively. Then,

$n(\mathcal{U}) = 800, n(C) = 224, n(H) = 240, n(B) = 336$

$n(H \cap B) = 64, n(B \cap C) = 80, n(H \cap C) = 40$

and, $n(H \cap B \cap C) = 24$

$\therefore$ Required number

$= n(C' \cap H' \cap B')$

$= n(C \cup H \cup B)'$

$= n(\mathcal{U}) - n(C \cup H \cup B)$

$= n(\mathcal{U}) - \{n(C) + n(H) + n(B) - n(C \cap H) - n(H \cap B) - n(B \cap C) + n(C \cap H \cap B)\}$

$= 800 - \{224 + 240 + 336 + 336 - 64 - 80 - 40 + 24\}$

$= 800 - 640 = 160$

Integer Answer Type

81 **(1)**

Let $\log_2 10 = p$ and $\log_5 10 = q$

Hence, $p + q = 1$

$x = p^3 + 3pq + q^3$

$= (p + q)^3 - 3pq(p + q) + 3pq$

$= 1 - 3pq + 3pq$

$= 1$

82 **(45)**

$2\tan^{-1}\frac{1}{5} = \tan^{-1}\left(\frac{\left(\frac{2}{5}\right)}{1 - \frac{1}{25}}\right) = \tan^{-1}\frac{5}{12}$

$\Longrightarrow 4\tan^{-1}\frac{1}{5} = 2\tan^{-1}\frac{5}{12}$

$= \tan^{-1}\left(\frac{\frac{5}{6}}{1 - \frac{25}{144}}\right) = \tan^{-1}\left(\frac{120}{119}\right)$

Given expression reduces to

$\tan^{-1}\frac{120}{119} + \tan^{-1}\frac{1}{99} - \tan^{-1}\frac{1}{70}$

$= \tan^{-1}\left(\frac{120}{119}\right) + \tan^{-1}\left[\frac{\frac{1}{99} - \frac{1}{70}}{1 + \frac{1}{(99)(70)}}\right]$

$$= \tan^{-1}\frac{120}{119} + \tan^{-1}\left(\frac{-29}{6931}\right)$$
$$= \tan^{-1}\left(\frac{120}{119}\right) - \tan^{-1}\left(\frac{1}{239}\right)$$
$$= \tan^{-1}\left(\frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119} \times \frac{1}{239}}\right)$$
$$= \tan^{-1}(1) = \frac{\pi}{4} \text{ radians} = 45 \text{ (in degrees)}$$

83 **(5)**

$$a = \frac{\log_5 175}{\log_5 245} = \frac{2 + \log_5 7}{1 + 2\log_5 7}$$

$$\Rightarrow a + 2a\log_5 7 = 2 + \log_5 7$$

$$\Rightarrow \log_5 7 = \frac{a-2}{1-2a} \quad \text{(i)}$$

$$b = \frac{\log_5 875}{\log_5 1715} = \frac{3 + \log_5 7}{1 + 3\log_5 7}$$

$$\Rightarrow b + 3b\log_5 7 = 3 + \log_5 7$$

$$\Rightarrow \log_5 7 = \frac{b-3}{1-3b} \quad \text{(ii)}$$

From Eqs. (i) and (ii); we get $\frac{a-2}{1-2a} = \frac{b-3}{1-3b} \Rightarrow \frac{1-ab}{a-b} = 5$

84 **(4)**

$$(2x^2 - 4.2^x + 4) + 1 + \big||b-1| - 3\big| = |\sin y|$$
$$\Rightarrow (2^x - 2)^2 + 1 + \big||b-1| - 3\big| = |\sin y|$$
$$\Rightarrow (2^x - 2)^2 + 1 + \big||b-1| - 3\big| = |\sin y|$$

LHS ≥ 1 and RHS ≤ 1

$\therefore 2^x = 2, |b-1| - 3 = 0$

$\Rightarrow (b-1) = \pm 3$

$\Rightarrow b = 4, -2$

85 **(1)**

$$f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$$
$$= \sin^2 x + \frac{1}{4}\left(\sin x + \sqrt{3}\cos x\right)^2 + \frac{1}{2}\cos x\left(\cos x - \sqrt{3}\sin x\right)$$
$$= \frac{5}{4}(\sin^2 x + \cos^2 x) = \frac{5}{4}$$

$(gof)x = g[f(x)]g(5/4) = 1$